Electric Current

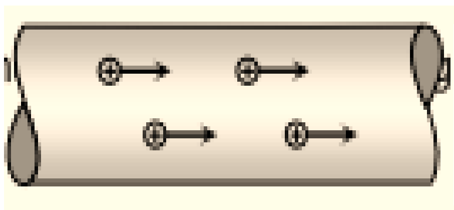
(1) The time rate of flow of charge through any cross section is called current. $i = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$

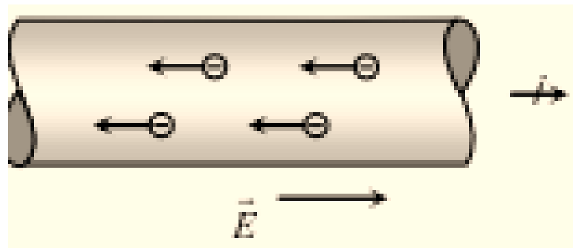
(2) If flow is uniform then $i = \frac{Q}{t}$

(3) Current is a scalar quantity. It's S.I. unit is ampere (A) and C.G.S. unit is emu and is called biot (Bi), or ab ampere. 1A = (1/10) Bi (ab amp.)

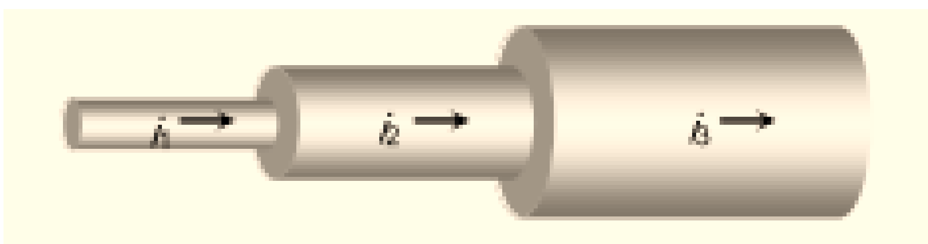
(4) Ampere of current means the flow of 6.25 x1018 electrons/sec through any cross-section of the conductor.

(5) The conventional direction of current is taken to be the direction of flow of positive charge, i.e. field and is opposite to the direction of flow of negative charge as shown below.





(6) For a given conductor current does not change with change in cross-sectional area. In the following figure $i_1 = i_2 = i_3$.



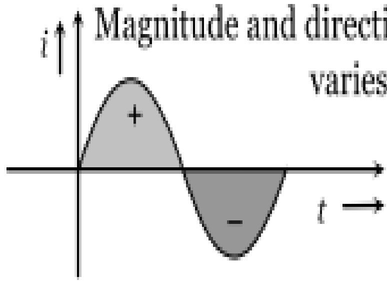
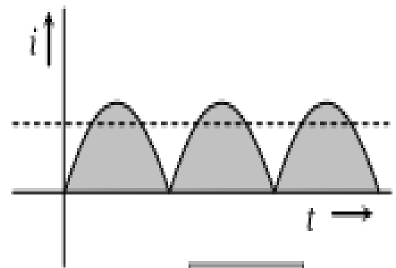
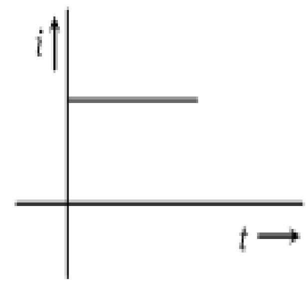
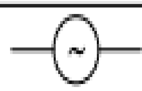
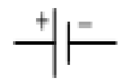
Current due to translatory motion of charge

- If n particle each having a charge q, pass through a given area in time t then $i = \frac{nq}{t}$

- If n particles each having a charge q pass per second per unit area, the current associated with cross-sectional area A is $i = nqA$

- If there are n particle per unit volume each having a charge q and moving with velocity v, the current thorough, cross section A is $i = nqvA$

Types of current

Alternating current (ac)	Direct current (dc)
(i)  Magnitude and direction both varies with time ac → Rectifier → dc (ii) Shows heating effect only	(i) (Pulsating dc) (Constant dc)   dc → Inverter → ac (ii) Shows heating effect, chemical effect and magnetic effect of current
(iii) It's symbol is 	(iii) It's symbol is 

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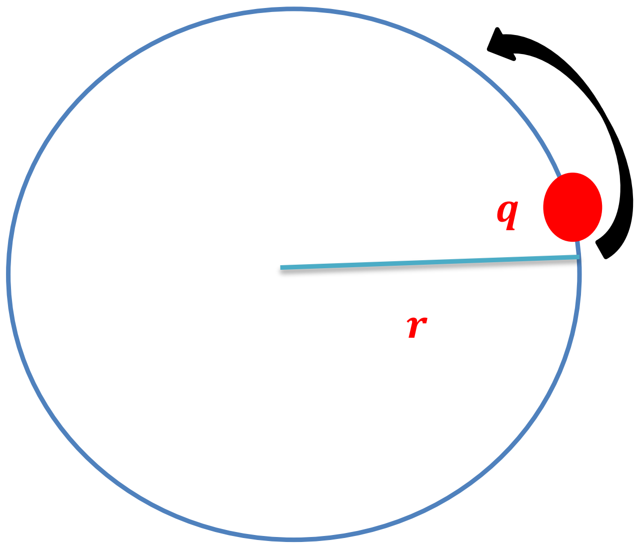
Current due to rotatory motion of charge

If a point charge q is moving in a circle of radius r with speed v (frequency (ν), angular speed(ω) and time period (T) then corresponding

$$i = q\nu = \frac{q}{T} = \frac{qV}{2\pi r} = \frac{q\omega}{2\pi}$$



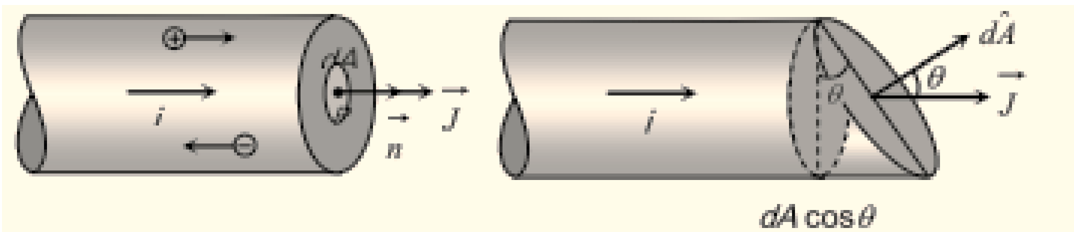
Current carriers

The charged particles whose flow in a definite direction constitutes the electric current are called current carriers. In different situation current carriers are different.

- **Solids :** In solid conductors like metals current carriers are free electrons.
- **Liquids:** In liquids current carriers are positive and negative ions.
- **Gases:** In gases current carriers are positive ions and free electrons.
- **Semi conductor** : In semi conductors current carriers are holes and free electron

Current Density (J)

Current density at any point inside a conductor is defined as a vector having magnitude equal to current per unit area surrounding that point. Remember area is normal to the direction of charge flow (or current passes) through that point.



- Current density at point P is given by $\vec{J} = \frac{di}{dA}\hat{n}$

- If the cross-sectional area is not normal to the current, but makes an angle θ with the direction of current then

$$J = \frac{di}{dACOS\theta}$$

Current Density (J)

- If current density J is uniform for a normal cross-section A then $\vec{J} = \frac{i}{\vec{A}}$

- Current density $\vec{J}$ is a vector quantity. It's direction is same as that of $\vec{E}$. It's S.I. unit is amp/m2 and dimension [L^{-2}A]

- In case of uniform flow of charge through a cross section normal to it as $i = nqvA$

  or, $\vec{J} = \frac{i}{\vec{A}} = nqv$

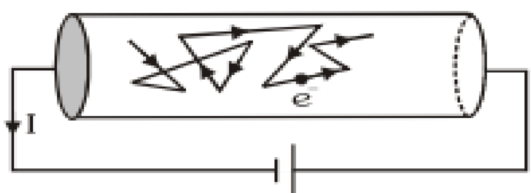
- Current density relates with electric field as $\vec{J} = \sigma \vec{E} = \frac{\vec{E}}{\rho}$

  Where, σ = electrical conductivity, ρ = resistivity

Drift Velocity

Drift velocity is the average uniform velocity acquired by free electrons inside a metal by the application of an electric field which is responsible for current through it.

- Drift velocity is very small it is of the order of 10^{-4}m/s as compared to thermal speed ~10^{5}m/s of electrons at room temperature.
- Drift velocity is also defined as the velocity with which the free electrons get drifted towards the positive terminal under the effect of the applied external electric field.



Under the action of electric field : Random motion of an electron with superimposed drift

- In addition to its thermal velocity, due to acceleration given by applied electric field, the electron acquires a velocity component in a direction opposite to the direction of the electric field. The gain in velocity due to the applied field is very small and is lost in the next collision

At any given time, an electron has a velocity $\overrightarrow{V_1} = \overrightarrow{u_1} + \vec{a}\,\tau_1$

Where, u_1= the thermal velocity and $\vec{a}\,\tau_1$= the velocity acquired by the electron under the influence of the applied electric field. τ_1= the time that has elapsed since the last collision. Similarly, the velocities of the other electrons are

$\overrightarrow{V_2} = \overrightarrow{u_2} + \vec{a}\,\tau_2$ $\overrightarrow{V_3} = \overrightarrow{u_3} + \vec{a}\,\tau_3$ .... $\overrightarrow{V_N} = \overrightarrow{u_N} + \vec{a}\,\tau_N$

The average velocity of all the free electrons in the conductor is equal to the drift velocity V_d the free electrons

$$\overrightarrow{V_d} = \frac{V_1+V_2+V_3+V_4\ldots\ldots+V_N}{N} = \frac{u_1+u_2+u_3+u_4\ldots\ldots+u_N}{N} + \vec{a}\left(\frac{\tau_1+\tau_2+\tau_3+\cdots+\tau_N}{N}\right)$$

Since, $\frac{u_1+u_2+u_3+u_4\ldots\ldots+u_N}{N} = 0$

So,

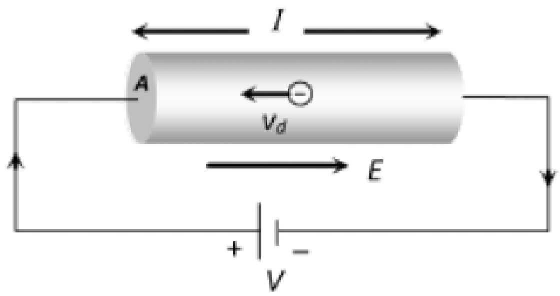
$$\overrightarrow{V_d} = \vec{a}\left(\frac{\tau_1+\tau_2+\tau_3+\cdots+\tau_N}{N}\right)$$

$$= \vec{a}\tau$$

$$= -\frac{e\vec{E}}{m}\tau$$

$$\boxed{\overrightarrow{V_d} = -\frac{e\vec{E}}{m}\tau}$$

Relation between current and drift velocity



Let, n = number density of free electrons and A= area of cross section of conductor.

Number of free electrons in conductor of length L = nAl

Total charge on these free electrons dq = neAl

Time taken by drifting electrons to cross conductor dt $= \frac{l}{v_d}$

Therefore, Current I$= \frac{dq}{dt} = \frac{\text{neAl}}{\frac{l}{v_d}} = \text{neA}v_d$

Relation between current density, conductivity and electric field

$I = neAv_d$

So, $\vec{J} = nev_d$

$= ne\frac{e\vec{E}}{m}\tau = \frac{ne^2\vec{E}}{m}\tau$

$= (\frac{ne^2\tau}{m})\ \vec{E}$

$= \sigma\, \boldsymbol{\vec{E}}$

Where, $\sigma = (\frac{ne^2\tau}{m})$

In vector form $\boldsymbol{\vec{J} = \sigma\, \vec{E} = \frac{\vec{E}}{\rho}}$ Ohm's law (at microscopic level)

Factors on which drift velocity depends

$$\overrightarrow{V_d} = \frac{i}{neA} = \frac{J}{ne} = \frac{\sigma \vec{E}}{ne} = \frac{\vec{E}}{\rho ne} = \frac{v}{\rho l ne}$$

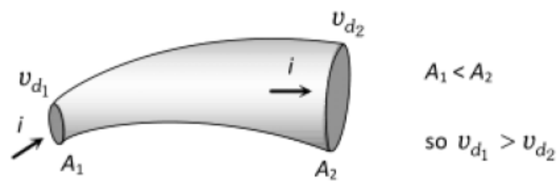
- The direction of drift velocity for electron in a metal is opposite to that of applied electric field (i.e. current density $\vec{J}$).

$$v_d \propto E$$

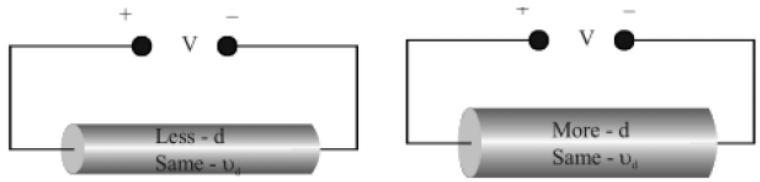
i.e., greater the electric field, larger will be the drift velocity.

- When a steady current flows through a conductor of non-uniform cross-section drift velocity varies inversely with area of cross-section

$$v_d \propto \frac{1}{A}$$



- If diameter (d) of a conductor is doubled, then drift velocity of electrons inside it will not change.



Relaxation time (τ)

The time interval between two successive collisions of electrons with the positive ions in the metallic lattice is defined as relaxation time.

$$\tau = \frac{mean\ free\ path}{r.m.s\ velocity\ of\ electrons} = \frac{\lambda}{v_{rms}}$$

So, With rise in temperature v_{rms} increases consequently τ decreases.

Mobility : Drift velocity per unit electric field is called mobility of electron, $\mu = \frac{v_d}{E}, It's\ unit\ is$ $m^2 volt^{-1} Sec^{-1}$

$Q_{1.}$ What will be the number of electron passing through a heater wire in one minute, if it carries a current of 8 A.

Solution

$$I = \frac{Ne}{t} \Rightarrow N = \frac{It}{e} = \frac{8 \times 60}{1.6 \times 10^{-19}} = 3 \times 10^{21} \text{ electrons}$$

Q_2 *A copper wire of diameter 1.02 mm carries a current of 1.7 amp. Find the drift velocity (v_d) of electrons in the wire. Given n, number density of electrons in copper $= 8.5 \times 10^{28}$ /m³.*

$I = 1.7\ A$

J = *current density*

$$= \frac{1}{\pi r^2} = \frac{1.7}{\pi \times (0.51 \times 10^{-3})^2}$$

$= nev_d$

$= 8.5 \times 10^{28} \times (1.6 \times 10^{-19}) \times v_d$

$$\therefore \quad v_d = \frac{1.7}{\pi \times (0.51 \times 10^{-3})^2 \times 8.5 \times 10^{28} \times 1.6 \times 10^{-19}}$$

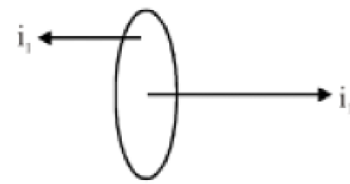
$= 1.5 \times 10^{-3}$ m/sec. $= 1.5$ mm/sec.

Q_3 *Find the electric current in a conductor (copper) of cross-section $A = 1nm^2$, conduction electron density $n = 8.69\times10^{28}/m^2$ and drift speed $v_d = 1$ cm /s.*

$i = nev_d A$
$= 8.69 \times 10^{28} \times 1.6 \times 10^{-19} \times 10^{-2} \times 1 \times 10^{-4}$
$= 8.69 \times 1.6 \times 10^{5}$ *amp*

Q_4 *n_1 electron/s passes through a given cross-section towards right with velocity v_1 and n_2 proton/s passes through the same cross-section with velocity v_2 in the same direction. Find the current through a given cross-sectional. Put $n_1 = 1.5 \times 10^{10}$ and $n_2 = 10^{10}$.*

$$i_1 = \frac{\Delta q}{\Delta r} = \frac{\Delta N_1 q_1}{\Delta r} = \frac{dN_1}{dt} q_1$$



$$i_2 = \frac{dN_2}{dt} q_2$$

$$i = i_1 + i_2$$

$$= \left(\frac{dN_1}{dt}\right)(-e) + \left(\frac{dN_2}{dt}\right)e$$

$i = (n_2 - n_1)\, e$
$= (1.5 \times 10^{10} - 1 \times 10^{10})\ 1.6 \times 10^{-19} = 0.5 \times 10^{-9}$ *amp*